MPOWER!
FOR KIDS WHO LIKE TO **SOLVE PROBLEMS**
ISSUE 3 | MAY 2016

ANSWERS

Bridges of Animal World
Kate is correct; it is not possible to cross each bridge exactly once.

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Magic Tree
1792 blossoms became butterflies.
Esmerelda picked 256 golden blossoms from the tree.

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A Probability Paradox

Problem 1. Suppose the elf has 100 wooden chests at the start of the game. What is the probability of winning the game, if you choose the best strategy?

Answer: If you stay with the chest you chose initially, your probability of winning will be 1/100. If you switch to the unopened chest, your probability of winning will be 99/100. Thus, it is much better to switch to the unopened chest!

Problem 2. What is the probability of getting at least one head after tossing a coin three times?

Answer: There are eight possible outcomes: (T, T, T), (H, T, T), (T, H, T), (T, T, H), (H, H, T), (H, T, H), (T, H, H), (H, H, H). Seven of the eight outcomes have at least one head, so the probability is 7/8.

Problem 3. If you roll two dice and add the numbers, what is the probability that the sum is 11? (Hint: Think how we get 11: 11 = 5 + 6 or 11 = 6 + 5, the first number is result of throwing of the first die and the second number is result of throwing of the second die.)

Answer: When we roll two dice, we have 36 possible outcomes: (1, 1), (1, 2), (1, 3), ..., (5, 6), (6, 6). There are two outcomes which have the sum equal to 11: (5, 6) or (6, 5). Thus, the probability is: 2/36 = 1/18.

Problem 4. If you roll a pair of dice and find the sum of the two numbers, which sum do you have a greater chance of getting: 2 or 11?

Answer: From problem 3 we know that the probability of getting a sum of 11 is 1/18. There is just one outcome for the sum 2: (1, 1). So, the probability of getting a sum of 2 is 1/36. Thus, you have a greater chance of getting a sum of 11.

Problem 5. What is the probability of getting doubles (for instance, 4 and 4) after throwing of a pair of dice?

Answer: We have 36 possible outcomes (see the solution of problem 3). There are 6 outcomes of doubles: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6). So, the probability is 6/36 = 1/6.

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M Challenge

1. 21

2. 9864

3. 4.5 units^2

4. Wednesday

5. 9

6. 35

7. $\frac{5}{4}n$

8. 19 times

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Amusements

1. n = 16

2. A = 1; B = 0

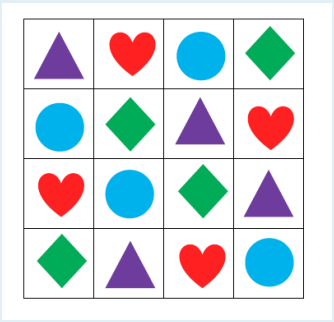
3. 20th term: $\frac{401}{20}$; nth term: $\frac{n^2+1}{20}$ (or $n + \frac{1}{n}$)

4. 5

5. 26 pounds

6. $\frac{1}{8}$ bushel

7.



8. $n\ \frac{1}{5}$

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The Binary System

Problem 4. A student says, "I am 1100 years old and I am a 111^{th} grader." Can you "translate" it?
Answer: The student says, "I am 12 years old and I am a 7^{th} grader." ($1100_2 = 12_{10}$ and $111_2 = 7_{10}$.)

Problem 5. Fill out an empty multiplication table for the binary system.
Answer:

	0	1	10	11	100
0	0	0	0	0	0
1	0	1	10	11	100
10	0	10	100	110	1000
11	0	11	110	1001	1100
100	0	100	1000	1100	10000

Problem 6. Is the number 10101010101010101_2 divisible by $6_{10} = 110_2$?
Answer: Because $110_2 = 6_{10}$ we check divisibility by 2 and by 3.

10101010101010101_2 is divisible by $2_{10} = 10_2$ and 10101010101010101_2 is divisible by $3_{10} = 11_2$ because the alternate sum of the number 10101010101010101_2 is 9_{10} and 9_{10} is divisible by 3_{10}.

Thus, 10101010101010101_2 is divisible by $110_2 = 6_{10}$.

Problem 7. For which natural number n is $2^n - 1$ divisible by 15?
Answer: We rewrite the number $(2^n - 1)_{10}$ in the binary system as: $(2^n - 1)_{10} = (10^n - 1)_2 = \underbrace{11 \ldots 1}_{\text{n ones}}{}_2$.

The number 15_{10} equals 1111_2. Consider four cases: n = 4k, n = 4k +1, n = 4k +2, n = 4k + 3.

a. If n = 4k, then $11 \ldots 1_2$ is divisible by 1111_2.
b. If n = 4k +1, then $\underbrace{11 \ldots 1}_{(4k+1)\text{ ones}}{}_2 = \underbrace{11 \ldots 10}_{4k\text{ ones}}{}_2 + 1_2$

   The number $11\ldots10_2$ is divisible by 1111_2 but the number 1_2 is not divisible by 1111_2.
   Thus, $11 \ldots 1_2$ is not divisible by 1111_2.
c. If n = 4k + 2, then, similarly to the case in (b), we get that $11 \ldots 1_2$ is not divisible by 1111_2.
d. If n = 4k + 3, then, similarly to the case in (b), we get that $11 \ldots 1_2$ is not divisible by 1111_2.

So $11 \ldots 1_2$ is divisible by 1111_2 if and only if n = 4k.

Thus, $2^n - 1$ is divisible by 15 if n = 4k.